

HOW TO USE OUR TALENTS BASED ON INFORMATION THEORY - OR SPENDING TIME WISELY

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ABSTRACT. We discuss the allocation of finite resources in the presence of a logarithmic diminishing return law, in analogy to some results from Information Theory. To exemplify the problem we assume that the proposed logarithmic law is applied to the problem of how to spend our time.

1. APTITUDE, TIME AND RESULTS

Resource allocation is a key problem in economy [1]. We present some considerations inspired by Information Theory [2,3] showing how, in general, whenever there is a diminishing return law of logarithmic type, the optimal allocation of the resources follows a water-filling behavior. To exemplify the treatment we analyze the economic problem of time allocation.

More precisely, the problem we want to address is the following: we have some resources (time, energy,...) which can be used for different possible activities, such as:

- 1: playing piano
- 2: horse racing
- 3: driving motorbikes
- ...
- N : working on new theorems.

We have a limited amount of resources - which we will indicate generally as *time*. The total time available is t_{tot} .¹

We are more skilled for some activities (they require us less *time* to give good results). We will then speak about the *aptitude* for the different activities (figure 1).

We assume that, as often in nature, the result is related to the time we spend on an activity, but in a less than proportional way. In other words, if we double the time we dedicate to an activity, the results we get will grow but less than twice (diminishing return). In particular, we assume that the results are described by a

¹This is the net time, after the effort we dedicate to mandatory activities.

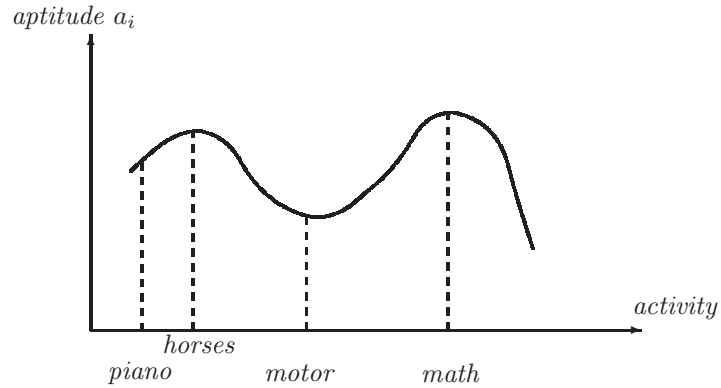


FIGURE 1. Aptitude.

logarithmic rule (figure 2):²

$$(1) \quad r_i = \log(1 + a_i t_i)$$

where, for the i -th activity, r_i represents the *result* (the higher, the better), a_i the *aptitude* we have for that activity, and t_i the *time* we dedicate to it. Note that, since $\log(1) = 0$, we get a result zero if we dedicate no time to an activity.

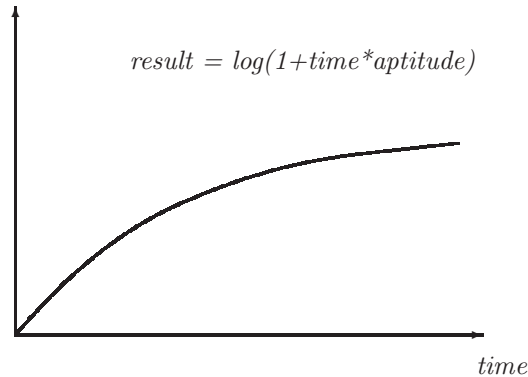


FIGURE 2. Result as a function of the spent time.

So, we have a set of aptitudes a_1, \dots, a_N , and, if we allocate the times t_1, \dots, t_N , we get the results r_1, \dots, r_N given by (1).

By spending the total time

$$(2) \quad t_{tot} = t_1 + t_2 + \dots + t_N$$

we get

$$(3) \quad r_{tot} = r_1 + r_2 + \dots + r_N.$$

²This logarithmic behavior is often encountered in Information Theory [2]: for example, by spending a power P , the amount of information that can be sent through a channel impaired by additive Gaussian noise (mutual information between the input and the output) is proportional to $\log(1 + P/N)$, where N is the noise power.

Now, the question is: how should we partition the total available time t_{tot} to get the maximum of the overall result r_{tot} ? Should we allocate more time to those activities where our skills are weaker? Or, should we spend more on those for which we have better aptitude?

Theorem 1. (*Resource allocation for logarithmic diminishing returns*)

In general, we must dedicate more time to the activities where we have the better aptitudes. Depending on the total available time, some of the activities where our aptitudes are worse must be abandoned.

More precisely, to optimally allocate the time, there exist a minimum aptitude a_{\min} such that:

- *all activities with $a_i < a_{\min}$ must be abandoned, i.e.:*

$$(4) \quad t_i = 0 \quad \text{if} \quad a_i \leq a_{\min}$$

- *for the activities with $a_i > a_{\min}$ we must allocate a time which is increasing with the aptitude, according to the rule*

$$(5) \quad t_i = \frac{1}{a_{\min}} - \frac{1}{a_i} > 0.$$

The threshold a_{\min} is the value that, used in (4) and (5), gives

$$t_1 + t_2 + \dots + t_N = t_{tot}.$$

Proof. (Analogous to [2]) The problem consists in finding the $t_i \geq 0$ such that the overall result is maximized, under the constraint $t_1 + \dots + t_N = t_{tot}$:

$$(6) \quad t_1, \dots, t_N = \arg \max_{t_1, \dots, t_N} \sum_{i=1}^N \log(1 + a_i t_i)$$

$$(7) \quad \text{with} \quad \sum_{i=1}^N t_i = t_{tot}$$

$$(8) \quad \text{and} \quad t_i \geq 0.$$

By using the Lagrange's multipliers method, we set

$$\frac{d}{dt_k} \left(\sum_{i=1}^N \log(1 + a_i t_i) - \lambda t_i \right) = 0$$

where λ is the multiplier. Then, we have

$$\frac{a_k}{1 + a_k t_k} - \lambda = 0$$

and therefore

$$\begin{aligned} t_k &= \frac{1}{\lambda} - \frac{1}{a_k} && \text{if greater than zero} \\ t_k &= 0 && \text{if} \quad a_k < \lambda \end{aligned}$$

The value of λ is that fulfilling (7).³ In the Theorem we indicate $a_{\min} = \lambda$. \square

³We can use the Karush-Kuhn-Tucker conditions to verify that the proposed solution maximizes the global result.

2. INTERPRETATION: WATER FILLING

Similarly to [2] for power allocation, the Theorem has a simple hydraulic interpretation.

In figure 3 we report the *inaptitude* ($1/a_i$) for the different activities. We obtain a container with an irregular bottom, where the deepest the bottom, the largest the aptitude.

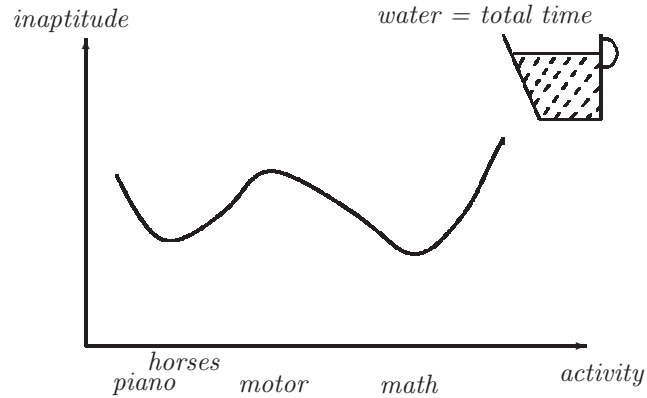


FIGURE 3. Inaptitude.

The total available time is represented by a certain amount of water. If we pour the water in the container, the depth of the water indicates the time to be dedicated to the activities.

The activities for which the inaptitude emerges above the water level must be abandoned. For the example of figure 4 it result that we should dedicate more time to horse-racing and math, while motorbike must be abandoned.

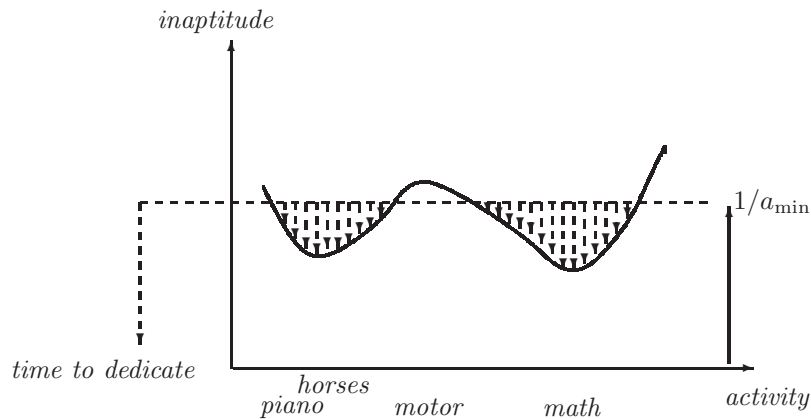


FIGURE 4. Optimal time partition by water-filling. The total quantity of water is the available total time. The depth of the water indicates the time to dedicate to the activities.

Therefore, if we have a lot of time we can cover all activities (but still the higher the aptitude for an activity, the larger the time we should dedicate to it). On the other extreme, if we have a small amount of total time we should invest it on the activities where we have the best aptitude. This is the case illustrated in figure 5.

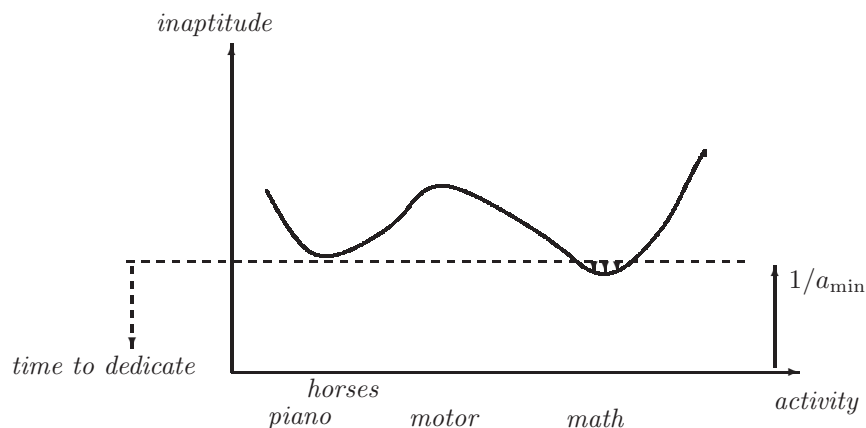


FIGURE 5. If the amount of time is small, we should better spend it on the activity where we have the best skills (in this case to math).

3. CONCLUSIONS

Assuming limited resources and a logarithmic relation between the product (aptitude*time) and the corresponding result, we should:

- abandon those activities for which the aptitude is below a certain threshold (the threshold depending on the total available time);
- spend more time for the activities where we have the better aptitudes;
- if the amount of available time is small, just dedicate it to the activity with the best aptitude.

REFERENCES

- [1] P. Dasgupta and G. Heal, *Economic theory and exhaustible resources*. Cambridge Univ Pr, 1980.
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- [3] T. A. Cover and J. A. Thomas, *Elements of Information Theory*, 1st ed. New York, NY, 10158: John Wiley & Sons, Inc., 1991.